Performance Analysis of Magnetic Resonance Image Denoising Using Contourlet Transform

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***Abstract*—A medical image denoising algorithm using contourlet transform is proposed and the performance of the proposed method is analysed with the existing methods. Noise in magnetic resonance imaging has a Rician distribution and unlike AWGN noise, Rician noise is signal dependent. Separating signal from Rician noise is a tedious task. The proposed approaches were compared with other transform methods such as wavelet thresholding and block DCT. Hard, soft and semi-soft thresholding techniques are described and applied to test images with threshold estimators like universal threshold. The results are compared based on the parameters: PSNR and MSE. Numerical results show that the contourlet transform can obtained higher PSNR than wavelet based and block DCT based denoising algorithms.**

***Index Terms*—Magnetic Resonance Image; denoising; contourlet transform.**

I. Introduction

Magnetic Resonance Imaging (MRI) [1] is a medical imaging technique that has proven to be valuable for examination of the soft tissues in the body. Because the resolution of MRI is very high, and the technology is essentially harmless, it has emerged as the most accurate and desirable imaging technology. MRI is primarily used to demonstrate pathological or other physiological alterations of living tissues. The time of MR imaging is limited by the patients comfort, non-stabilities and artifacts of the tomography system, and physical limitations during dynamic applications such as heart imaging or functional MRI. At present, fast methods of magnetic resonance are used, which allows a significant reduction of investigation time. Retrieved images have a low signal-to-noise ratio (SNR) and a small contrast. Therefore, noise reduction techniques are of great interest in MR imaging. As the simplest and fast technology, Gaussian filter has been applied extensively in MRI pre-processing. But this kind of lowpass filter suffers from removing high-frequency signal components as well as noise. The drawback of such digital image filtering technique is the reduction of sharpness, resolution, and image contrast. The methods typically used are based on an analysis of wavelet coefficients[2] and contourlet coefficients [3] [4] [5] which allows decomposing the input image into several frequency sub-bands and separating the noise part from useful image information. This paper proposes a method for image denoising[6] using contourlet transform and a comparative study of different denoising techniques like wavelet transform and block DCT [7].

Perhaps one of the most interesting result in wavelet research is the connection between the wavelet transform in harmonic analysis and filter banks in discrete signal processing. The connection was set up using the multiresolution analysis by Mallat [8] and Meyer. Such a connection allows the wavelet transform, initially defined in the continuous domain, to be computed with fast algorithms based on filter banks. The multiresolution analysis provides a natural framework for the understanding of wavelet bases, and starting from iterated filter banks leads to the famous construction of compactly supported wavelets by Daubechies.

Section II describes the existing methods using wavelet transform and block DCT. Proposed MR image denoising method using contourlet transform is discussed in section III. Section IV deals with the performance evaluation of the proposed and existing methods and Section V describes the simulation results of proposed methods and comparison with the existing methods. Finally section VI concludes the paper.

II. Existing Methods

A. Denoising of MRI using Wavelet transform

Continuous wavelet transform of a square integrable function $f(t)$ is given in (1).

$$W(a,b)=\int_{-\infty}^{\infty} f(t)\psi^{*}_{a,b}(t)dt \tag{1}$$

where a and b are real constants and $*$ denotes complex conjugation.

Thresholding involves the reduction or complete removal of selected wavelet coefficients in order to separate out the noise within the signal. The thresholding method[9], used in the wavelet based de-noising technique, distinguishes between insignificant coefficients due to the noise of magnetic resonance device and significant coefficients consisting of signal components. The wavelet coefficients with a value lower than a particular threshold, T correspond to noisy samples and they can be avoided which leads to noise reduction in the

978-0-7695-5033-6/13 $26.00 © 2013 IEEE
DOI 10.1109/ICACC.2013.84

image domain. Two basic thresholding techniques used are hard and soft thresholding[10]. In hard thresholding technique, the wavelet coefficients that are lower than threshold value T are cancelled and the remaining coefficients are unaffected as given in (2)

$$\hat{C}(i) = \begin{cases} C(i) & , \quad |C(i)| \geq T \\ 0 & , \quad |C(i)| < T \end{cases} \tag{2}$$

The soft thresholding technique cancels the wavelet coefficients that are lower than threshold T, but it tries to isolate signal from noise in the remaining coefficients as in (3)

$$\hat{C}(i) = \begin{cases} \text{sign}[C(i)][|C(i)| - T] & , \quad |C(i)| \geq T \\ 0 & , \quad |C(i)| < T \end{cases} \tag{3}$$

The most important part of the de-noising algorithm is the estimation of the optimal threshold value. When the threshold value is too low, then noise reduction is inefficient. On the other hand, when it is too high, then detailed image information can be lost. In the proposed work, the most efficient estimation algorithm, universal thresholding defined as (4) is used.

$$T = \sigma_{est}\sqrt{2\log(N)} \tag{4}$$

where N is the number of input image pixels, represents the standard deviation of noise, which can be estimated by the Donoho and Johnstone theorem as statistical median of all detailed wavelet coefficients from the first decomposition level divided by the constant 0.6745 as in (5).

$$\sigma_{est} = \frac{\text{MAD } (C_D^1)}{0.6745} \tag{5}$$

B. Denoising of MRI using Block DCT

The discrete cosine transform of a list of n real numbers $s(x)$, $x = 0, ..., n-1$, is the list of length n given by (10).

$$S(u) = \sqrt{\frac{2}{n}}C(u)\sum_{x=0}^{n-1} s(x)\cos\frac{(2x+1)u\pi}{2n} \quad u = 0, \ldots, n \tag{6}$$

$$C(u) = \begin{cases} 2^{-\frac{1}{2}} & , \quad \text{for } u = 0 \\ 1 & , \quad \text{otherwise} \end{cases} \tag{7}$$

Each element of the transformed list $S(u)$ is the inner (dot) product of the input list $s(x)$ and a basis vector. The constant factors are chosen so that the basis vectors are orthogonal and normalized.

The one-dimensional DCT is useful in processing 1D signals such as speech waveforms. For analysis of two-dimensional (2D) signals such as images, we need a 2D version of the DCT. For an $n \times m$ matrix s, the 2D DCT is computed in a simple way: The 1D DCT is applied to each row of s and then to each column of the result. Thus, the transform of s is given as in (8).

$$\begin{aligned} S(u,v) = \quad & \tfrac{2}{\sqrt{mn}}C(u)C(v)\sum_{y=0}^{m-1}\sum_{x=0}^{n-1} s(x,y)\cos\tfrac{(2x+1)u\pi}{2n} \\ & \cos\tfrac{(2y+1)v\pi}{2m} \quad u = 0, \ldots, n \quad v = 0, \ldots, m \end{aligned} \tag{8}$$

where

$$C(u) = \begin{cases} \frac{1}{\sqrt{2}} & , \quad \text{for } u = 0 \\ 1 & , \quad \text{otherwise} \end{cases} \tag{9}$$

The discrete cosine transform (DCT) is a technique for converting a signal into elementary frequency components. It is widely used in image compression. A new denoising technique is proposed to remove the Rician noise from magnetic resonance images using block DCT transform. To this point, we have defined functions to compute the DCT of a list of length N = 8 and the 2D DCT of an 8 x 8 array. We have restricted our attention to this case partly for simplicity of exposition, and partly because when it is used for image compression, the DCT is typically restricted to this size. Rather than taking the transformation of the image as a whole, the DCT is applied separately to 8 x 8 blocks of the image. We call this a blocked DCT. To compute a blocked DCT, we do not actually have to divide the image into blocks. Since the 2D DCT is separable, we can partition each row into lists of length 8, apply the DCT to them, rejoin the resulting lists, and then transpose the whole image and the process is repeated.

It is well know that shift-sensitivity of the DWT can be dramatically improved by using a dual-tree DWT, an over complete expansion that is redundant by a factor of 2. And the undecimated DWT is a shift-invariant discrete transform which has an expansion-factor of $\log N$. Inspired by redundant wavelets, the proposed method use a redundant DCT coefficients. An image with N pixels is transform to N $m \times m$ blocks with DCT. The estimate of the noise-free image x is obtained using a hard thresholding method to these DCT blocks. The local denoised estimate at block $j \times j$ is obtained using (10) - (12).

$$c_j = H(y_j) \tag{10}$$

$$\hat{c}_j = T(c_j, \tau) \tag{11}$$

$$\hat{x}_j = H^{-1}(\hat{c}_j) \tag{12}$$

where H is a 2D DCT, c_j are the transform coefficients of the j block, and T is the hard thresholding operator. Then all estimates are combined from all overlapping j blocks at position i using the weighted average as in (13)

$$\hat{x}(i) = \frac{\sum\limits_{j=1}^{V} \theta_j \hat{x}_j(i)}{\sum\limits_{j=1}^{V} \theta_j} \tag{13}$$

$$\theta_j = \frac{1}{1 + |C_j^{dc}|} \tag{14}$$

where V is the number of blocks relational to and is the weight of each block j, which is proportional to the inverse of the direct component of DCT. The function can also be form like:

$$\hat{C}_j(i) = \begin{cases} 0 & , \quad \text{if } |p_j(i)| < \sigma \\ c_j(i) & , \quad \text{otherwise} \end{cases} \tag{15}$$

which compares the magnitude of the noise-free DCT coefficients with the standard deviation of the noise. So the above 2D DCT filter is used to get a pre-filtered image as an approximation of the noise-free image.

III. Proposed Method

A. Denoising of MRI using Contourlet Transform

The contourlet transform offers a flexible multiresolution and directional decomposition for images, since it allows for a different number of directions at each scale. For the contourlet transform to satisfy the anisotropy scaling law, as in the curvelet transform, we simply need to impose that the number of directions is doubled at every other finer scale of the pyramid. The contourlet transform is almost critically sampled, with a small redundancy factor of up to 1:33. Comparing this with a much larger redundancy ratio of the discrete implementation of the curvelet transform , the contourlet transform is much more suitable for image compression. Furthermore, the contourlet transform can be designed to be a tight frame, which implies robustness against the noise due to quantization or thresholding.

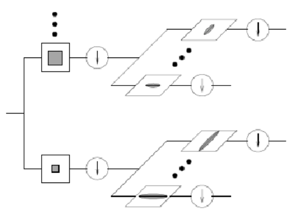

Fig. 1. Process of taking Contourlet transform

In Counterlet Transform, image is decomposed by a double filter-bank structure[11], where the first one captures the edge points and the second one links these edge points into contour segments as shown in Fig. 1. The gray areas in the boxes represent the support sizes of the filters. Laplacian pyramid scheme is shown in Fig. 2. The outputs are a coarse approximation c and a difference d between the original signal and the prediction. The process can be iterated by decomposing the coarse version repeatedly. The proposed reconstruction scheme is also shown in Fig. 2.

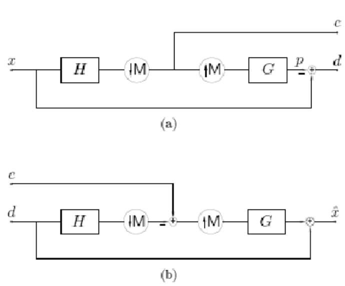

Fig. 2. Laplacian pyramid scheme. (a) Analysis scheme (b) The proposed reconstruction scheme

One way of achieving a multiscale decomposition is to use a Laplacian pyramid (LP) as introduced by Burt and Adelson. The LP decomposition at each step generates a sampled lowpass version of the original and the difference between the original and the prediction, resulting in a bandpass image. The process can be iterated on the coarse version.

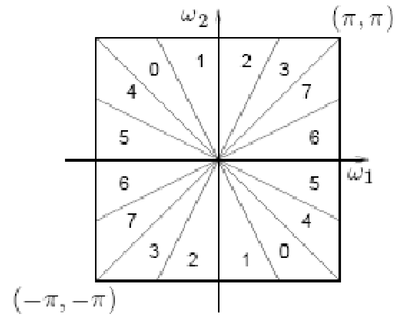

Fig. 3. Directional filter bank frequency partitioning where l= 3 and there are $2^3 = 8$ real wedge-shaped frequency bands

In 1992, Bamberger and Smith introduced a 2-D directional filter bank (DFB) that can be maximally decimated while achieving perfect reconstruction. The DFB is efficiently implemented via a l-level tree-structured decomposition that leads to 2l sub bands with wedge-shaped frequency partition as shown in Fig. 3.

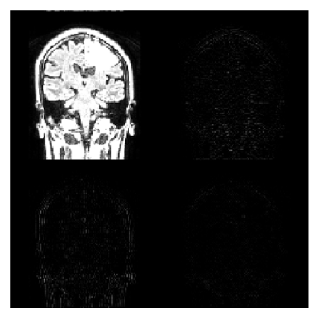

Fig. 4. Contourlet decomposition of brain MR Image

In the recent years, Do and Vetterli proposed a multiscale and multidirectional image representation method named contourlet transform, which can effectively capture image edges and contours. The contourlet transform is constructed by Laplacian pyramid (LP) and directional filter banks (DFB) The Laplacian Pyramid at each level generates a Low pass output (LL) and a Band pass output (LH, HL, and HH). The Band pass output is then passed into Directional Filter Bank, which results in contourlet coefficients. The Low pass output is again passed through the Laplacian Pyramid to obtain more coefficients and this is done till the fine details of the image are obtained. Fig. 4 shows the decomposition of brain MR Image using contourlet transform.

The parameters which are utilized for contourlet image denoising includes noise variance which is estimated using

Fig. 5. Simulation results (a) MR Image, (b) Noisy MR Image, (c) Denoised image using block DCT, (d) Denoised image using wavelet and (e) Denoised image using contourlet.

the mean absolute deviation (MAD) method as in (16).

$$\sigma_n^2 = \left(\frac{\text{median}(c_{i,j})}{0.6745}\right)^2 \tag{16}$$

where $c_{i,j}$ is the contourlet coefficients of noisy image. The threshold T for the contourlet coefficients of noisy image is given by :

$$T = \frac{3}{4}N\left(\frac{\sigma_n^2}{\sigma_g}\right) \tag{17}$$

where N is total number of pixels in the image and σ_g is the standard deviation of the noisy image. The performance of the proposed method is compared with the hard threshold, soft threshold and Wiener filter in the wavelet domain using PSNR as

$$\text{PSNR} = 10\log_{10}\frac{255^2}{\text{MSE}} \tag{18}$$

where MSE denotes the mean square error for two $m \times n$ images, where one of the images is considered a noisy approximation of the other and is given as :

$$\text{MSE} = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left[I(i,j) - \hat{I}(i,j)\right]^2 \tag{19}$$

IV. Performance Evaluation

To get the measure of the performance of various denoising techniques[12], the experimental results are evaluated according to two criteria: MSE and PSNR [13], [14]. These parameter decides which methods & thresholding technique gives best result. MSE is the cumulative square of the difference between two images (original image and denoised image) and can be calculated using (20).

$$\text{MSE} = \frac{1}{N^2}\sum_{i,j=0}^{N-1}\|C(i,j) - \hat{C}(i,j)\|^2 \tag{20}$$

$$\text{PSNR} = 20\log\left(\frac{2^B - 1}{\text{MSE}}\right)dB \tag{21}$$

V. Simulation Results

Simulations are done with random noise added to the MR image. Biomedical images especially MRI is susceptible to noise, which has effect on its interpretation and understanding. Fig. 5 shows that proposed contourlet based denoising algorithm has better performance compared to the existing methods. Performance evaluation is done using MSE and PSNR. Table I shows the performance analysis for different methods.

TABLE I
THE MSE AND PSNR RESULTS OF MRI BRAIN IMAGE

Parameters Measured	Transform Used		
	Block DCT	Wavelet	Contourlet
MSE	244	168	167
PSNR of Denoised Image (dB)	24.23	25.86	25.88

VI. Conclusion

A medical image denoising algorithm using contourlet transform with directional filter banks and Laplacian pyramid is proposed and the performance of the proposed method is analysed with the existing methods of denoising using wavelet transform and block DCT. Simulation results shows that contourlet transform has better denoising capabilities compared to existing methods.

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