A Generic Endoatmospheric Guidance based on Hybrid Approach

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Abstract- A generic endoatmospheric guidance based on hybrid approach has been proposed in this paper. The problem here is to guide the vehicle in a fuel optimum maneuver from an initial state to the desired final state. The algorithm is based on the method of regular perturbation which is useful for the development of real time guidance algorithms. The proposed algorithm is suitable for both ascent and entry missions and here the ascent guidance problem alone is addressed. Hybrid method combines the numerical and analytic approaches thereby making it advantageous when compared to each of the methods being used independently. The results obtained validate the supremacy of hybrid method over conventional methods.

Keywords – ***hybrid problem; regular perturbation; endoatmospheric guidance***

I. INTRODUCTION

Traditional launch vehicle guidance employs open loop guidance in the atmospheric phase and closed loop guidance once the vehicle is sufficiently out of the atmosphere [1]. Hanson et al made a first effort to compare closed loop and open loop guidance within the atmosphere [2]. Though the guidance approaches for exo-atmospheric phase are efficient and reliable, the use of open loop guidance for the atmospheric phase flight has been one cause of costly launch delays. This occurs when actual wind profile differs significantly from the mean profile used in computing the attitude control program. To overcome this problem a closed loop guidance approach using the hybrid method is developed for the endoatmospheric flight phase.

Numerical approaches when used alone have the disadvantage that the solution will take a long time to converge [3]. To be useful as a feedback guidance algorithm the solutions should converge quickly and reliably at each instant when the solution is updated during flight.

On the other hand the analytic approaches use regular perturbation method [4], [5] to correct the atmospheric guidance. In this method an approximate solution to a problem is constructed in terms of a small parameter which is termed as the expansion parameter. The advantage of using this method is that the non-linearities can be treated as perturbations which can be neglected in the formulation of the zeroth order problem. This significantly simplifies the problem and makes it possible to obtain a closed form solution. The problem here is that significant non-linearties like aerodynamics must be neglected in the zeroth order problem in order to obtain analytic solution. Higher-order terms of the expansion include the effects of the neglected perturbation dynamics. This technique is preferred as a real-time, on-line guidance scheme to alternative numerical iterative optimization schemes because of the unreliable convergence properties of these iterative guidance schemes [6]. As the guidance algorithm is implemented in real time, closed form non-iterative solutions are desired. So optimal control approach is used which results in two point boundary value problem with split boundary conditions [7].

Calise et al includes the construction of optimal guidance law based on asymptotic expansion with a small expansion parameter. The methods of Regular Perturbation Analysis and Collocation for numerical solution for optimal control problem are combined in [8].

In this paper, a generic endoatmospheric guidance algorithm based on hybrid approach has been developed. Hybrid method developed here, uses Calculus of Variations to solve the analytic portion of the problem and Method of Variation of Extremals to solve the numerical part whereas Calise et al used the Method of Collocation to find the numerical solution. The algorithm developed is compatible for both ascent and re-entry missions. Here the ascent problem alone is addressed.

This paper is organized as follows. Section II gives a brief outline of the ascent guidance problem. Section III and IV presents the concepts of perturbation theory and the Regular Perturbation Approach. The detailed ascent guidance problem formulation and zeroth order problem are discussed in sections V and VI. The first order problem and the hybrid problem are described in sections VII and VIII. Results obtained using the proposed algorithm is presented in section IX and conclusions in section X.

II.ASCENT GUIDANCE PROBLEM

The problem here is to guide the vehicle in a fuel optimum maneuver from an initial state to the desired final state and the vehicle is acted upon by thrust force, aerodynamics and gravitational force.

The problem is formulated as a Calculus of Variations problem where the aim is to minimize the fuel consumption thereby maximizing the payload into the orbit. Since here aerodynamics is also accounted, the problem cannot be solved analytically. For this purpose, analytically tractable portion of the problem is found, i.e. with the exclusion of aerodynamics and chosen a small angle approximation for the steering angle, alpha (α). As Calculus of Variations could not find a solution for the entire problem, here the Method of Regular Perturbation is used to find the solution.

III.PERTURBATION THEORY

One of the main uses of asymptotic analysis is to provide approximations to differential equations that cannot easily be solved explicitly. The following is a general 2^{nd} order differential equation for y(*x,* λ), a function of x and λ,

$$\frac{d^2y}{dx^2} + p(x,\lambda)\frac{dy}{dx} + q(x,\lambda)y = r(x,\lambda) \qquad (1)$$

The independent variable here is x, with respect to which all differentiation and integration is applied, λ and any other variables upon which the solution of y could depend on are known as physical parameters and no differentiation or integration is carried out with respect to them.

The variable with respect to which the asymptotic behavior is studied is known as the asymptotic variable. In classical asymptotic analysis the asymptotic variable is taken as the independent variable of the differential equation. In perturbation theory the asymptotic behavior is studied with respect to a small physical parameter, usually denoted by ε.

The point in the domain around which the asymptotic behavior is studied is known as the asymptotic accumulation point. The most common differential equation problems where approximations are sought for are those of perturbation theory, where the accumulation point is $\varepsilon = 0$.

Perturbation theory deals with problems that contain a small parameter conventionally denoted by ε and solutions are sought as ε approaches 0. Perturbation theory can be split into regular and singular forms. Here Regular Perturbation theory alone is discussed.

IV. REGULAR PERTURBATION METHOD

Very often, a mathematical problem cannot be solved exactly or, if the exact solution is available, it exhibits such an intricate dependency in the parameters that it is hard to use as such. It may be the case, however, that a parameter can be identified, say ε, such that the solution is available and reasonably simple for ε= 0. Then, this solution can be altered for non-zero but very small values of ε, say ε<1. This forms the basis of regular perturbation theory. In regular perturbation problem, the solution of a problem is sought as an expansion in terms of the asymptotic sequences $\{1, \varepsilon, \varepsilon^2 ...\}$ as $\varepsilon \to 0$.

A regular perturbation problem is one for which the perturbed problem for small, nonzero values of ε is qualitatively the same as the unperturbed problem for ε = 0. One typically obtains a convergent expansion of the solution with respect to ε, consisting of the unperturbed solution and higher-order corrections.

The simple quadratic problem given here containing *ε* as a coefficient of *x* is an example of a perturbation problem.

$$x^2 - 1 = \varepsilon x \qquad (2)$$

The two roots of this equation are

$$x_1 = \varepsilon/2 + \sqrt{1 + \varepsilon^2/4}\,;\, x_2 = \varepsilon/2 - \sqrt{1 + \varepsilon^2/4} \qquad (3)$$

For small ε, these roots are well approximated by the first few terms of their Taylor series expansion

$x_1 = 1+ \varepsilon /2 + \varepsilon^2/8 + O(\varepsilon^3)$; (4a)
$x_2 = -1+ \varepsilon /2 + \varepsilon^2/8 + O(\varepsilon^3)$; (4b)

The problem described here is solved using regular perturbation theory which involves four steps.

STEP A
Assume that the solution(s) of (2) can be Taylor expanded in ε. Then x is represented as
$x = X_0 +\varepsilon X_1 +\varepsilon^2 X_2 + O(\varepsilon^3)$; (5)
for X_0, X_1, X_2 to be determined.

STEP B
Substitute (5) into (2) written as $x^2 -1 - \varepsilon x = 0$, and expand the left hand side of the resulting equation in power series of ε . Using
$x^2 = X_0^2+2 \varepsilon X_0 X_1 + \varepsilon^2(X_1^2 +2 X_0 X_2)+ O(\varepsilon^3)$; (6a)
$\varepsilon x =\varepsilon X_0 +\varepsilon^2 X_1+ O(\varepsilon^3)$; (6b)
which gives
$X_0^2 -1+ \varepsilon^1 (2X_0X_1 - X_0) + \varepsilon^2 (X_1^2 +2 X_0X_2 - X_1) + O(\varepsilon^3) = 0$; (7)

STEP C
Equate to zero the successive terms of the series in the left hand side of (7):
$O(\varepsilon^0) : X_0^2-1 = 0$; (8a)
$O(\varepsilon^1) : 2 X_0 X_1 - X_0 = 0$; (8b)
$O(\varepsilon^2) : X_1^2 +2 X_0 X_2- X_1 = 0$; (8c)
$O(\varepsilon^3)$: ....

STEP D
Successively solve the sequence of equations obtained in (8). Since $X_0^2 - 1 = 0$ has two roots, $X_0 = \pm1$, one obtains

$X_0 = 1; X_1 = 1/2; X_2 =1/8$; (9a)
$X_0 = -1; X_1 = 1/2; X_2 =1/8$; (9b)

It can be checked that substituting (9) into (5) recovers (4).

V. MATHEMATICAL MODELING OF THE ASCENT GUIDANCE PROBLEM

The problem model and the associated state variables are defined in figure 1. The position and velocity vectors in the guidance co-ordinate frame are represented as (r, Φ, u, w).

The vehicle is acted upon by thrust force, aerodynamics and gravitational force. The planar equations of motion of a vehicle with spherical gravity model [9] are considered here.

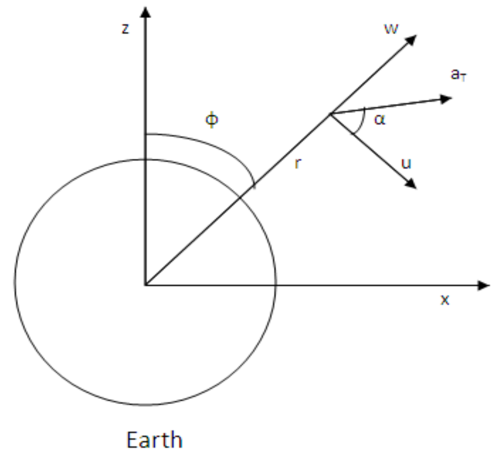

Figure 1: Problem Model

Initial Conditions of state variables as obtained from navigation are

$$r(t_0) = r_0, w(t_0) = w_0 u(t_0) = u_0,\ \phi(t_0) = \phi_0 \quad (10)$$

Final Conditions are

$$r(t_f) = r_f,\ w(t_f) = w_f,\ u(t_f) = u_f \quad (11)$$
$$\phi(t_f) \ and\ t_f\ are\ free$$

where r is the radial distance, w is the vertical component of velocity, u is the horizontal component of velocity, Φ is the range angle ,t_0 is the initial time and t_f is the final time.

The Flat Earth (FE) Guidance equations of motion are given by

$$\dot{r} = -w \quad (12)$$
$$\dot{w} = a_T \sin(\alpha) - (\mu/r^2 - u^2/r) \quad (13)$$
$$\dot{u} = a_T \cos(\alpha) - \frac{uw}{r} \quad (14)$$
$$\dot{\phi} = u / r \quad (15)$$

where α is the steering angle, μ is the gravitational constant, and a_T is the thrust acceleration.

The assumptions taken are given by equations (16)-(19).

$$\sin(\alpha) \approx \alpha \approx \tan(\alpha) \quad (16)$$

$$\cos(\alpha) \approx 1 \tag{17}$$

$$\frac{uw}{r} = 0 \tag{18}$$

$$(\mu / r^2 - u^2 / r) = g \tag{19}$$

$$a_T = \frac{T}{m_0 - \dot{m}t} = \frac{\frac{T}{\dot{m}}}{m_o / \dot{m} - t} = \frac{v_e}{\tau - t} \tag{20}$$

Equations (16) and (17) indicates small angle approximation for α, uw/r is the Coriolis force, g is the gravitational acceleration, T is the thrust, m_0 is the initial mass of fuel, $\dot{m}$ is the mass flow rate, v_e is the exhaust velocity, τ is the vehicle fuel constant and t is the time.

The steering angle, alpha takes the form

$$\tan(\alpha) = at + b, \tag{21}$$

where a and b are the steering parameters.

Taking the effect of aerodynamics into account and resolving lift and drag forces into Cartesian co-ordinate frame results in the following equations of motion

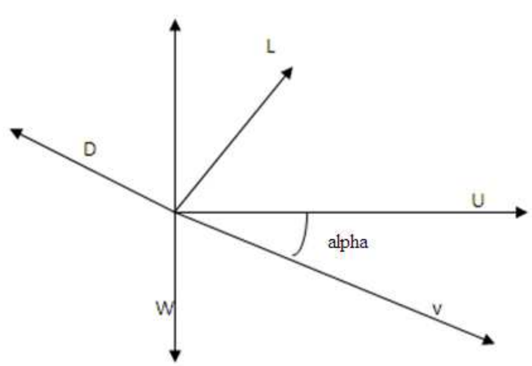

Figure 2: Transformation into Cartesian co-ordinate frame

$$\dot{r} = -w \tag{22}$$

$$\dot{w} = \frac{v_e}{\tau - t}\sin(\alpha) + g - (\frac{(Lu + Dw)}{v}) \tag{23}$$

$$\dot{u} = \frac{v_e}{\tau - t}\cos(\alpha) + (\frac{(Lw - Du)}{v}) \tag{24}$$

The aerodynamic forces Lift and Drag are defined as

$$L = qSC_L \tag{25}$$

$$D = qSC_D \tag{26}$$

where C_L and C_D are the lift and drag coefficients respectively which have fixed values, S is the reference area, and is q the dynamic pressure given by q=0.5ρv^2.

The density is assumed to be of the form

$$\rho = \rho_0 e^{-\beta h}, \tag{27}$$

where h is the atmospheric scale height, ρ_0 is the reference density and β is the scale height.

v is given by $\sqrt{u^2 + w^2}$ (28)

The steering parameters [9] a and b are computed as,

$$\begin{bmatrix} a \\ b \end{bmatrix} = M^{-1} \begin{bmatrix} \Delta r \\ \Delta w \end{bmatrix} \tag{29}$$

where,

$$\begin{bmatrix} \Delta r \\ \Delta w \end{bmatrix} = \begin{pmatrix} -r_f + r_o + \frac{1}{2}g(t_f^2 - t_0^2) + w_o t_0 - w_f t_f \\ w_o - w_f + g(t_f - t_0) \end{pmatrix} \tag{30}$$

$$M = \begin{pmatrix} \frac{1}{2}v_e(t_f^2 - t_0^2) + \tau[v_e(t_f - t_0) + \tau(u_o - u_f)] & v_e(t_f - t_0) + \tau(u_o - u_f) \\ v_e(t_f - t_0) + \tau(u_o - u_f) & u_o - u_f \end{pmatrix} \tag{31}$$

The final time t_f is computed as

$$t_f = t_0 + (\tau - t_0)\left(1 - \exp\left(\frac{u_o - u_f}{v_e}\right)\right) \tag{32}$$

When t_{go} becomes small, i.e. as t approaches t_f, the steering angle is calculated using

$$\alpha = -\tan^{-1}\left[\frac{(w_0 - w_f) + g(t_f - t_o)}{u_f - u_0}\right] \tag{33}$$

Now the problem is a Nonlinear Two point Boundary Value Problem (TPBVP) which cannot be solved analytically. Numerical Integration poses difficulties since the boundary conditions are split. In order to obtain optimal trajectory, nonlinear TPBVP needs to be solved. Here comes the application of regular perturbation method. As explained earlier an expansion parameter ε is introduced which results in the following equations of motion

$$\dot{r} = -w \tag{34}$$

$$\dot{w} = \frac{v_e}{\tau - t}\sin(\alpha) + g - \varepsilon\left(\frac{(Lu + Dw)}{v}\right) \tag{35}$$

$$\dot{u} = \frac{v_e}{\tau - t}\cos(\alpha) + \varepsilon\left(\frac{(Lw - Du)}{v}\right) \tag{36}$$

This separation of the dynamics into primary and perturbation terms allows a closed form solution to be obtained.

In both the zeroth and first order problems the vertical dynamics alone is considered.

VI. ZEROTH ORDER PROBLEM

Using the method of regular perturbation [5] the state variables r and w are expressed as

$$r = r_0 + \varepsilon r_1 + \varepsilon^2 r_2 + \ldots\ldots \tag{37}$$

$$w = w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + \ldots\ldots \tag{38}$$

Since for the zero order problem ε=0, the equations of motion are re-written as

$$\dot{r}_0 = -w_0 \tag{39}$$

$$\dot{w}_0 = \frac{v_e}{\tau - t}\sin(\alpha) + g \tag{40}$$

VII. FIRST ORDER PROBLEM

To obtain first order solution, substitute equations (37) to (38) in equations (34) to (35) respectively and make ε=1. Here final time, t_f is not constrained. Then the equations of motion become

$$\dot{r}_1 = -w_1 \tag{41}$$

$$\dot{w}_1 = \frac{1}{2}\rho_0 S.e^{-\beta(r_0 - r_e)}(-C_d.w_0^2) \tag{42}$$

VIII. HYBRID PROBLEM

In hybrid method both the zero and first order problems are combined as given in equations (43)-(44). Here zeroth order problem is solved analytically and first order problem is solved numerically. Combining the solution obtained using the two methods, final hybrid solution is obtained. Here ε is taken as 1.

$$\dot{r} = -w \tag{43}$$

$$\dot{w} = \frac{v_e}{\tau - t}\alpha + g + \varepsilon\left[\frac{v_e}{\tau - t}(\sin(\alpha) - \alpha) + \frac{1}{2}\rho_0 S.e^{-\beta(r_0 - r_e)}(-C_d.w_0^2)\right] \tag{44}$$

IX. SIMULATION RESULTS

The result of the hybrid problem along with a comparison of this method with the analytic method is presented in this section. The initial conditions are taken as w_0=-500 m/sec, r_0=6448000 m. The terminal constraints to be satisfied are r_f =6610000 m, w_f =-1600 m/sec.

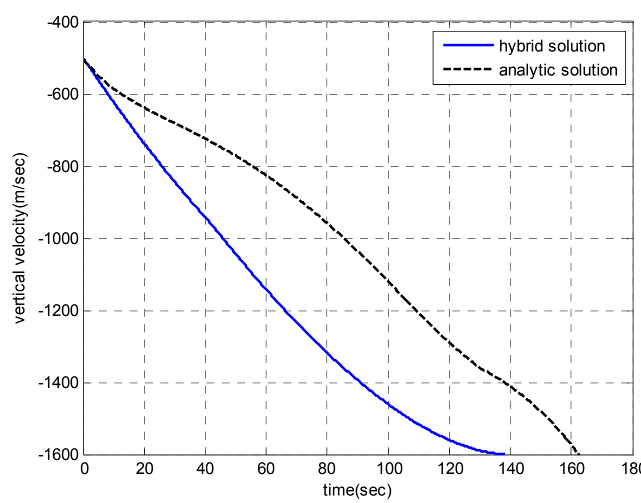

Figure 3: Vertical velocity Vs time

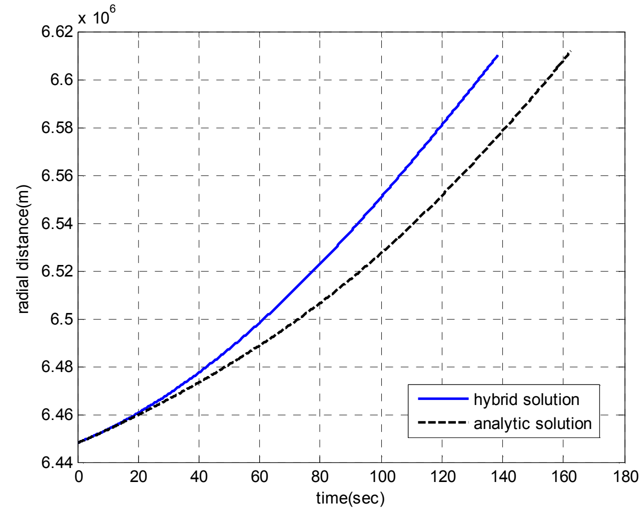

Figure 4: Radial distance Vs time

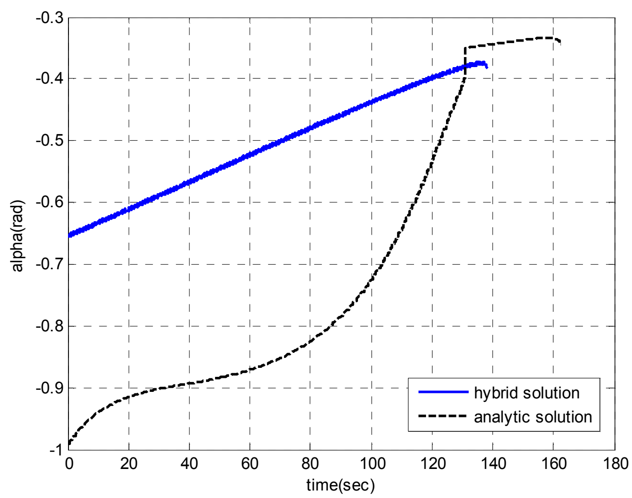

Figure 5: Steering angle Vs time

The results in figures 3 and 4 indicate that the state variables vertical velocity and radial distance are converging for a final time of 162.37 sec for the analytic method whereas a much faster convergence is obtained in the case of hybrid method which is 138.4 sec. Figure 5 shows the variation of steering angle (α) with time for the two cases.

X. CONCLUSIONS

An endoatmospheric guidance algorithm which has the flexibility for performing for both ascent and re-entry missions is developed. The equations of motion for the ascent guidance problem are formulated in terms of a small expansion parameter, ε. The forces acting on the vehicle are separated into dominant forces and perturbation forces. In order to solve the guidance problem, the zeroth order problem is formulated by equating the expansion parameter to zero. Once the solution to the zeroth order problem is obtained, the higher order correction terms can be included to compensate for the effect of perturbation forces which are neglected in the zeroth order problem, thus formulating the first order problem. The zeroth and first order problems are combined using expansion parameter ε, and this result in hybrid problem, whose solution shows its supremacy over conventional methods.

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