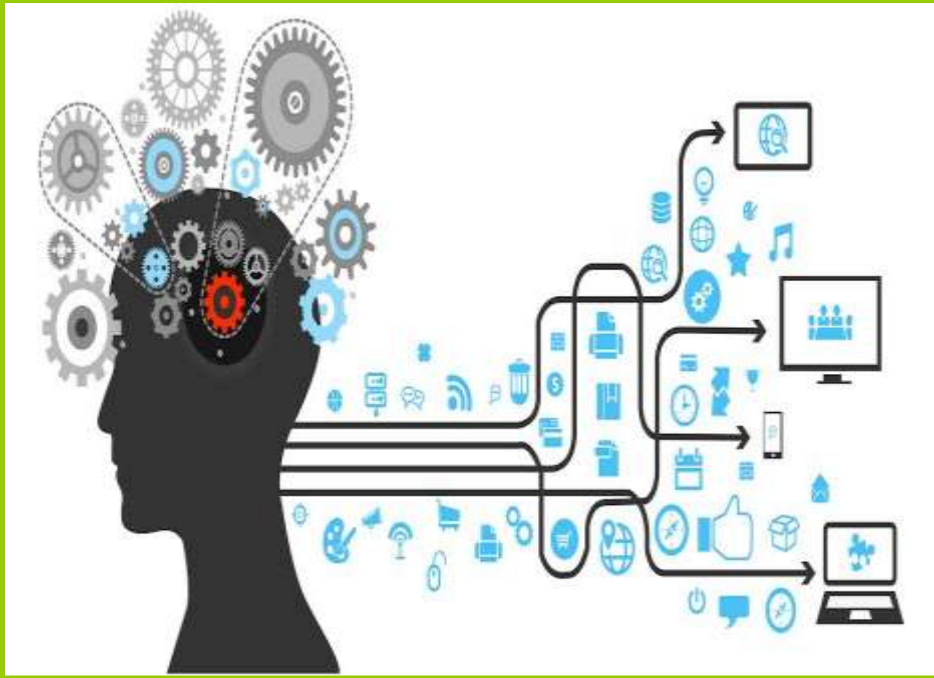
EE486 Soft Computing



Dr. Unnikrishnan P.C.
Professor, EEE

Module II

- Fuzzy Rules and Fuzzy Reasoning

Crisp Proposition

- A **proposition** is a statement which acquires a truth value. **(either *true or false*).**

  Ex: P: Water Boils at 90℃

  Q: Sky is Blue

- Classical logic deals with **propositions**
- To represent complex information we need a sequence of propositions linked using **connectives**
- The *propositional logic handles combination of logical variables.*

Example of *propositional logic*

The **conjunction** of the two sentences:

Grass is green

Pigs don't fly

is the sentence:

Grass is green and pigs don't fly

The conjunction of two sentences will be true if, and only if, each of the two sentences from which it was formed is true.

Symbols for Connective

ASSERTION	P					"P IS TRUE"
NEGATION	¬P	~	!		NOT	"P IS FALSE"
CONJUCTION	P^Q	.	&	&&	AND	"BOTH P AND Q ARE TRUE
DISJUNCTION	PvQ	||	|		OR	" EITHER P OR Q IS TRUE"
IMPLICATION	P->Q		⇒		IF...THEN	"IF P IS TRUE THEN Q IS TRUE."
EQUIVALENCE	P⇔ Q	=	⇔		IF AND ONLY IF	"P AND Q ARE EITHER BOTH TRUE OR FALSE"

Truth Value

- The truth value of a statement is truth or falsity.
    - p is either true or false
    - ~p is either true of false
    - p^q is either true or false, and so on.
- Truth table is a convenient way of showing relationship between several propositions.

Truth Table of Negation

	P	~P
Case 1	T	F
Case 2	F	T

As you can see "P" is a true statement then its negation "~P" or "not P" is false.

If "P" is false, then "~P" is true.

Truth Table of Conjunction

	P	Q	P ∧ Q
Case 1	T	T	T
Case 2	T	F	F
Case 3	F	T	F
Case 4	F	F	F

Truth Table of Disjunction

	P	Q	P V Q
Case 1	T	T	T
Case 2	T	F	T
Case 3	F	T	T
Case 4	F	F	F

Logical Implication

Logical implication is a type of relationship between two statements or sentences.

The relation translates verbally into "logically implies" or "if/then" and is symbolized by a double-lined arrow pointing toward the right ($\Rightarrow$).

If A and B represent statements, then A $\Rightarrow$ B means "A implies B" or "If A, then B." The word "implies" is used in the strongest possible sense.

Logical Implication …….

As an example of logical implication, suppose the sentences A and B are assigned as follows:

A = The sky is overcast.
B = The sun is not visible.

In this instance, $A \Rightarrow B$ is a true statement (assuming we are at the surface of the earth, below the cloud layer.)

However, the statement $B \Rightarrow A$ is not necessarily true; it might be a clear night. Logical implication does not work both ways.

Logical Implication …….

However, the sense of logical implication is reversed if both statements are negated. That is,

$(A \Rightarrow B) \Rightarrow (-B \Rightarrow -A)$

Using the above sentences as examples, we can say that if the sun is visible, then the sky is not overcast. This is always true. In fact, the two statements AB and –B$\Rightarrow$-A are logically equivalent. $\Rightarrow$

Truth Table of Logical Implication

	P	Q	$P \Rightarrow Q$
Case 1	T	T	T
Case 2	T	F	F
Case 3	F	T	T
Case 4	F	F	T

Truth Table of Logical Implication

P	Q	P⇒Q
T	T	T
T	F	F
F	T	T
F	F	T

Consider the following example:

"If you pass soft computing, then I'll give you a chocolate."

The statement will be *true* if I keep my promise and *false* if I don't.

- Suppose it's *true* that you pass SC and it's *true* that I give you a chocolate. Since I kept my promise, the implication is *true*.
- Suppose it's *true* that you pass SC but it's *false* that I give you a chocolate. Since I *didn't* keep my promise, the implication is *false*.
- What if it's false that you pass SC ? Whether or not I give you a chocolate, I haven't broken my promise. Thus, the implication can't be false, so it must be true.

Truth Table of Equivalence

	P	Q	$P \Leftrightarrow Q$
Case 1	T	T	T
Case 2	T	F	F
Case 3	F	T	F
Case 4	F	F	T

Example. Construct a truth table for the formula $\sim P \wedge (P \Rightarrow Q)$

P	Q	$\sim P$	$P \Rightarrow Q$	$\sim P \wedge (P \Rightarrow Q)$
T	T			
T	F			
F	T			
F	F			

Example. Construct a truth table for the formula $\sim P \wedge (P \Rightarrow Q)$

P	Q	$\sim P$	$P \Rightarrow Q$	$\sim P \wedge (P \Rightarrow Q)$
T	T	F		
T	F	F		
F	T	T		
F	F	T		

Example. Construct a truth table for the formula $\sim P \wedge (P \Rightarrow Q)$

P	Q	~P	$P \Rightarrow Q$	$\sim P \wedge (P \Rightarrow Q)$
T	T	F	T	
T	F	F	F	
F	T	T	T	
F	F	T	T	

Example. Construct a truth table for the formula $\sim P \wedge (P \Rightarrow Q)$

P	Q	~P	P⇒Q	~P∧(P⇒Q)
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Tautology

- **Tautology** is a proposition formed by combining other proposition (p,q,r…) which is **always true** regardless of truth or falsehood of p,q,r…

- The opposite of a tautology is a **contradiction**, a formula which is **"always false".** In other words, a contradiction is false for every assignment of truth values to its simple components

Example. Show that $(P \Rightarrow Q) \vee (Q \Rightarrow P)$ is a tautology.

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \vee (Q \Rightarrow P)$
T	T			
T	F			
F	T			
F	F			

Example. Show that $(P \Rightarrow Q) \vee (Q \Rightarrow P)$ is a tautology.

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \vee (Q \Rightarrow P)$
T	T	T		
T	F	F		
F	T	T		
F	F	T		

Example. Show that $(P \Rightarrow Q) \vee (Q \Rightarrow P)$ is a tautology.

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \vee (Q \Rightarrow P)$
T	T	T	T	
T	F	F	T	
F	T	T	F	
F	F	T	T	

Example. Show that $(P \Rightarrow Q) \vee (Q \Rightarrow P)$ is a tautology.

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \vee (Q \Rightarrow P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

The last column contains only T's. Therefore, the formula is a **tautology**.

Example. Construct a truth table for $(P \Rightarrow Q) \wedge (Q \Rightarrow R)$

- If the '$\Rightarrow$' connective deals with two different universes of discourse, i.e. $A \subset X$ and $B \subset Y$ where X and Y are two universes of discourse then '$\Rightarrow$' connective is represented by the relation

$$R = (A \times B) \cup (\bar{A} \times Y)$$

- In such a case, $P \Rightarrow Q$ is linguistically referred to as

IF x is A THEN y is B

- The compound proposition for

IF x is A THEN y is B ELSE y is C

Where P,Q,S are defined by sets A,B,C, $A \subset X$ and $B \subset Y$ and $C \subset Y$

IF x is A THEN y is B $(P \Rightarrow Q)$

IF x is ~A THEN y is C $(\sim P \Rightarrow S)$

Compound proposition: $(P \Rightarrow Q) \vee (\sim P \Rightarrow S)$

Corresponding Relation R

$$R = (A \times B) \cup (\bar{A} \times C)$$


Thank you

Example. Construct a truth table for $(P \Rightarrow Q) \wedge (Q \Rightarrow R)$

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T